# Mark Scheme (Results)

January 2014

Pearson Edexcel International GCSE
Further Pure Mathematics (4PM0/02)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$, where $|pq|=|c|$ leading to $x=....$

$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for $a$, $b$ and $c$, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

4. Use of calculator:

In most cases a calculator may be used. The M mark can only be awarded if **both** roots are seen to be correct. This applies even in cases where other methods gain the M mark when only one root is required.

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

**Use of a formula:**

Generally, the method mark is gained by

**either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

| Question Number | Answer | Marks |
| --- | --- | --- |
| **1** | gradient of $AB = \frac{9-3}{5-9} = -\frac{3}{2}$ oe | B1 |
| | gradient of perp $= \frac{2}{3}$ | B1ft |
| | coords of midpoint of $AB$ are $(7,6)$ | B1 |
| | Equation of perp bisector: $y-6=\frac{2}{3}(x-7)$ | M1 (must use grad of $\perp$ and coords of midpoint) |
| | $2x-3y+4=0$ or multiple | A1 [5] |

**Notes**

B1 for the (correct) gradient of $AB$

B1ft for the gradient of the perpendicular, ft ie give for $-\frac{1}{\text{their gradient of } AB}$

B1 for both coordinates of the midpoint of $AB$

M1 for any complete method for the equation of the perpendicular bisector. Their gradient of the perpendicular and their coordinates of the midpoint must be used.

A1 for $2x-3y+4=0$ or any integer multiple of this (inc negative multiples). A correct equation in the form ...= 0, even if the $y$ term is shown first.

| Question Number | Answer | Marks |
|---|---|---|
| **2** | $V=\frac{1}{3}\pi r^2 h=\frac{1}{12}\pi h^3$ | B1 |
| | $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{4}\pi h^2$ | M1 |
| | $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{4}{\pi h^2}\times 12$ | M1A1ft |
| | $=\frac{4}{16\pi}=\frac{3}{\pi}$ cm/s | A1 |
| | (Or work with *r* instead of *h* at start) | [5] |

**Notes**

B1 for obtaining a correct unsimplified expression for *V* in terms of a single variable.

$V=\frac{1}{12}\pi h^3$ or $V=\frac{2}{3}\pi r^3$

M1 for attempting the differentiation of *V* wrt their chosen variable (*h* or *r*)

M1 for a correct relevant chain rule expression or expressions which can lead to $\frac{\mathrm{d}h}{\mathrm{d}t}$ ie

$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ or $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ used with $\frac{\mathrm{d}h}{\mathrm{d}t}=2\frac{\mathrm{d}r}{\mathrm{d}t}$

Allow a chain rule written $\frac{\mathrm{d}t}{\mathrm{d}h}$

A1ft for substituting *their* $\frac{\mathrm{d}h}{\mathrm{d}V}$ or $\frac{\mathrm{d}r}{\mathrm{d}V}$ (algebraic sufficient for this mark) **and** $\frac{\mathrm{d}V}{\mathrm{d}t}=12$.

Must be $\frac{\mathrm{d}h}{\mathrm{d}t}$ now.

A1cao using $h=4$ or $r=2$ to obtain $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{3}{\pi}$ (cm/s) (Accept any equivalent **exact** fraction). Ignore decimals following a correct exact answer.

| Question Number | Answer | | Marks |
|---|---|---|---|
| **3** | $y=\frac{1}{5}(22-6x)$ | OR $5x^2+5xy-15x=10$ <br> $5y=22-6x$ | M1 |
| | $x^2+\frac{x}{5}(22-6x)-3x=2$ | $5x^2+x(22-6x)-15x=10$ | M1 |
| | $x^2-7x+10=0$ | | A1 |
| | $(x-2)(x-5)=0$ | | M1 |
| | $x=2,\quad x=5$ | | A1 |
| | $x=2\Rightarrow y=2,\quad x=5\Rightarrow\ y=-\frac{8}{5}$ | | A1 |
| | | | [6] |

**Notes**

M1 for re-arranging the linear equation to read $y=$ .... or $x=$ ....OR multiplying the quadratic by 5 so the linear can be substituted without re-arrangement

M1dep for substituting to obtain a quadratic in a single variable (either $y$ or $x$)

A1 for a correct 3 term quadratic. Need not have 0 on one side.

$x^2-7x+10=0$ or $5y^2+2y-16=0$ or any equivalent

M1dep for solving their quadratic by any valid means inc calculator (see initial notes)

Dependent on both previous M marks.

A1 for any 2 correct values, can be both $x$, both $y$ or a pair consisting of one of each

A1 for the other 2 correct values

| Question Number | Answer | Marks |
|---|---|---|
| 4 | **<u>Penalise only once in the question for non 3 sf answers.</u>** | |
| (a) | $\frac{1}{2}\times 10^2 \sin\theta = 20$ | M1 |
| | $\theta = 0.4115... = 0.412$<br>(Any complete method M1; Correct answer A1) | A1 (2) |
| (b) | $r\theta = 10\times 0.412 = 4.12$ | M1A1ft (2) |
| (c) | area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}\times 100\times 0.4115 = 20.5754$ | M1 |
| | shaded area = $20.5754 - 20 = 0.576$ (accept 0.575) | M1A1 (3)<br>[7] |

**Notes**

M1 for using Area of $\Delta = \frac{1}{2}ab\sin C$ with $a = b = 10$ and $A = 20$

A1cao for $\theta = 0.412$ **must be 3 sf and in radians**

There are longer methods. Give M1 if any complete method is used and A1 for correct value.

(b)

M1 for using length arc $= r\theta$ or any other valid method. Some may work in degrees - allow M1 for correct formula (for degrees) used with angle in degree

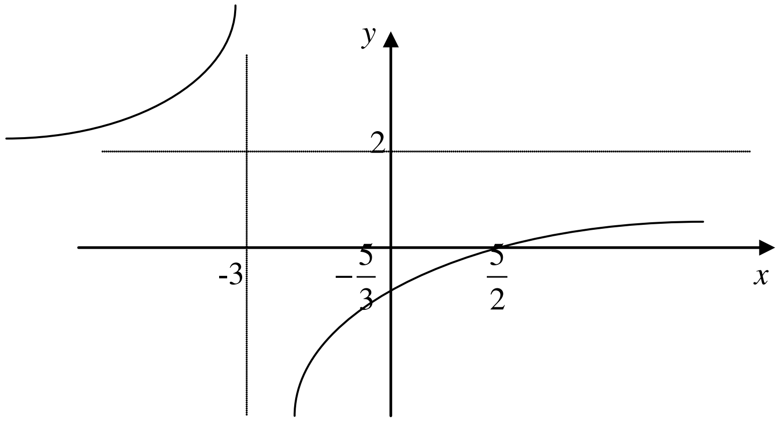
A1cao for $r\theta = 4.12$ If rounding penalised in (a), award for more figures, see initial notes for info on rounding/truncating penalties

(c)

M1 for using $A = \frac{1}{2}r^2\theta$ with *their* $\theta$ and $r = 10$ Some may work in degrees - allow M1 for correct formula (for degrees) used with angle in degrees

M1 for *their* area of the sector - 20. Must be this way round even if it gives a negative answer.

A1cao . For 0.576 or 0.575. Answer must be 3 sf (or more if already penalised)

| Question Number | Answer | Marks |
| --- | --- | --- |
| **5** **(a)** | (i) $y = 2$ | B1 |
| | (ii) $x = -3$ | B1 (2) |
| **(b)** | (i) $\left(\frac{5}{2}, 0\right)$ accept $x = \frac{5}{2}$ | B1 |
| | (ii) $\left(0, -\frac{5}{3}\right)$ accept $y = -\frac{5}{3}$ oe (accept -1.67 or better) | B1 (2) |
| **(c)** |  | B1(2 branches in corr quads.)<br><br>B1 (asymptotes)<br><br>B1(Crossing points) (3) |
| **(d)** | $\frac{dy}{dx} = \frac{2(x+3)-(2x-5)}{(x+3)^2}$ (or divide and differentiate) | M1A1 |
| | $x = -1$ $\frac{dy}{dx} = \frac{2\times 2-(-7)}{2^2} = \frac{11}{4}$ oe | A1ft (3) |
| | | [10] |

**Notes**

(a)

(i) B1 for $y = 2$ must be an equation

(ii) B1 for $x = -3$ must be an equation

**NB:** If correct equations seen but incorrectly identified, award B1B0

(ie (i) $x = -3$ etc)

(b)

(i) B1 for $\left(\frac{5}{2},0\right)$ or $x=\frac{5}{2}$

(ii) B1 for $\left(0,-\frac{5}{3}\right)$ accept $y=-\frac{5}{3}$ oe (accept -1.67 or better)

**NB:** As in (a), correct answers the wrong way round gain B1B0

(c) **NB:** No follow through here.

B1 for two branches in the correct "quadrants"

B1 for the asymptotes. The lines should have their equations or the coords of the points where they cross the axes shown. The curve should approach the asymptotes but not touch/cross either (or both) lines nor should it clearly bend away from an asymptote.

B1 for indicating the coordinates of the points where the curve crosses the axes.

**NB:** A candidate who draws one branch can score B0B1B1; A candidate who introduces extra crossing points scores B1B1B0 max.

(d)

M1 for attempting the differentiation of the curve equation. If quotient rule used, the denominator must be squared and the 2 numerator terms must be subtracted (but their order may be incorrect). If division used first, diff of $-11(x+3)^{-1}$ may be seen instead of use of quotient rule

A1 for a correct differential $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x+3)-(2x-5)}{(x+3)^2}$ or $11(x+3)^{-2}$ oe

A1ft for a correct numerical value when $x=-1$ follow through their differential.

| Question Number | Answer | Marks |
| --- | --- | --- |
| **6** **(a)** | $V=(80-2x)(40-2x)x$ | M1A1 |
| | $V=3200x-240x^2+4x^3$ * | A1 (3) |
| **(b)** | $\frac{dV}{dx}=3200-480x+12x^2$ | M1 |
| | $\frac{dV}{dx}=0 \quad 12x^2-480x+3200=0$<br>$3x^2-120x+800=0$ | M1dep |
| | $x=\frac{120\pm\sqrt{120^2-12\times800}}{6}$ | M1dep |
| | $x=31.54...(\text{not poss.})$ or $8.452...=8.45$ | A1 |
| | $\frac{d^2V}{dx^2}=-480+24x$ | M1 |
| | $x=8.45\Rightarrow\frac{d^2V}{dx^2}<0 \therefore \text{max.}$ | A1 (6) |
| **(c)** | $V_{max}=3200\times8.452-240\times8.452^2+4\times8.452^3$ or $(80-2\times8.452)(\ )$ etc | M1 |
| | $V_{max}=12300$ | A1 (2)<br>[11] |

**Notes**

(a)

M1 for attempting the volume of the box, must be dimensionally correct

A1 for all three lengths correct

A1cso for $V=3200x-240x^2+4x^3$ *

(b)

M1 for differentiating the **given** expression for $V$

M1dep for equating their differential to 0

M1dep for solving the resulting 3 term quadratic. See general principles.

A1 for $x = 8.45$ **must be 3 sf** (31.54 need not be shown (unless calculator solution); if included, a choice must be made now or later)

M1 for attempting the second differential of $V$

A1cso for establishing a max. Award this mark only if the expression for the second differential is algebraically correct and a correct value of $x$ has been obtained. No need to evaluate (ignore incorrect evaluations if that is the only error). A conclusion must be seen.

*Alternatives for the last 2 marks:*

(i)M1 for taking values of $x$ near to and on either side of their $x$ **and** calculating the numerical values of $\frac{dV}{dx}$ for both of these values.

A1 for all work correct and a correct conclusion.

(ii)M1 for taking values of $x$ near to and on either side of their $x$ **and** calculating the numerical values of $V$ for both of these values and for their $x$.

A1 for all work correct and a correct conclusion.

If this method used, marks for (c) can be given also.

(iii) By considering the curve. Evidence that it is a cubic, through the origin, and $V$ is negative when $x$ is negative (M1). Hence max at lesser of the roots of quadratic ie at $x = 8.45$ (A1)

(c)

M1 for taking their value of $x$ and substituting in the **given** expression for $V$

A1 for $V = 12300$ **must be 3 sf**, but deduction may have been made in (b).

| Question Number | Answer | Marks |
|---|---|---|
| **7** **(a)** | Ht. of $\Delta AEB = 3\text{ cm}$ | B1 |
| | $PG^2 = 13^2 + 15^2 \quad PG = 19.849... = 19.8\text{ cm}$ | M1A1 (3) |
| **(b)** | $AC^2 = 13^2 + 4^2, \quad AC = 13.601.... = 13.6\text{ cm}$ | M1,A1 (2) |
| **(c)** | $\sin\theta = \frac{13}{19.84}$ or $\cos\theta = \frac{15}{19.84}$ or $\tan\theta = \frac{13}{15}$ | M1A1ft |
| | $\theta = 40.9°$ | A1 (3) |
| **(d)** | $\sin\varphi = \frac{13}{13.6}$ or $\cos\varphi = \frac{4}{13.6}$ or $\tan\varphi = \frac{13}{4}$ | M1A1ft |
| | $\varphi = 72.9$ | A1 (3) [11] |

**Notes**

In this question penalise once for failing to round to 3 sf **and** once for failing to round to 0.1°

(a) B1 for height of $\Delta AEB = 3$ (cm)

M1 for correct use of Pythagoras to find $PG$ ie $15^2 + (10 + \text{their ht})^2$

A1cao for $PG = 19.8$ (cm) **must be 3 sf**

(b)

M1 for correct use of Pythagoras, using their height of $\Delta AEB$ (or any other complete method)

A1cao for 13.6(cm) **must be 3 sf unless already penalised**

(c)

M1 for using a trig function to find the correct angle (Alt, use cosine rule)

A1ft for correct numbers in their trig function (or cosine rule), follow through on previously calculated lengths

A1cao for 40.9° **must be to 1 dp**

(d)

M1 for using a trig function to find the correct angle

A1ft for correct numbers in their trig function, follow through on previously calculated lengths

A1cao for 72.9° or 107.1° **must be to 1 dp unless already penalised**

| Question Number | Answer | Marks |
| --- | --- | --- |
| **8** **(a)** | $AB = 2OA \Rightarrow OC = 3OA$ | |
| | $\overrightarrow{OC} = 3(\mathbf{a}+\mathbf{e}) \Rightarrow \overrightarrow{AB} = 2(\mathbf{a}+\mathbf{e})$ | M1A1 (2) |
| **(b)** | $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OE} = -2(\mathbf{a}+\mathbf{e}) - \mathbf{a} + \mathbf{e} = -(3\mathbf{a}+\mathbf{e})$ | M1,A1 (2) |
| **(c)** | $\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC} = \frac{3}{5}\times 2(\mathbf{a}+\mathbf{e}) + \mathbf{e}, = \frac{6}{5}\mathbf{a} + \frac{11}{5}\mathbf{e}$ | M1A1,A1 (3) |
| **(d)** | $\overrightarrow{PQ} = k\overrightarrow{PC} = \frac{k}{5}(6\mathbf{a}+11\mathbf{e})$ | |
| | $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \mathbf{a} + \frac{2}{5}\times 2(\mathbf{a}+\mathbf{e}) + \frac{k}{5}(6\mathbf{a}+11\mathbf{e})$ | M1A1 |
| | $\overrightarrow{OQ} = \overrightarrow{OE} + \overrightarrow{EQ} = \mathbf{e} + p(\mathbf{a}+\mathbf{e})$ | B1 |
| | $\therefore \frac{1}{5}(9+6k)\mathbf{a} + \frac{1}{5}(4+11k)\mathbf{e} = (1+p)\mathbf{e} + p\mathbf{a}$ | |
| | $\frac{1}{5}(9+6k) = p \quad \frac{1}{5}(4+11k) = 1+p$ | M1 |
| | Eliminate $p$ to obtain $k = 2$ or eliminate $k$ to obtain $p = \frac{21}{5}$ | A1 |
| | $\therefore \overrightarrow{OQ} = \frac{21}{5}\mathbf{a} + \frac{26}{5}\mathbf{e} \quad \lambda = \frac{21}{5},\ \mu = \frac{26}{5}$ | A1 (6) |
| | | [13] |

**Notes**

**a, e** need not be bold or written $\underset{\sim}{a}$ in students' work but $\overrightarrow{AB}$ etc must have the vector arrows when referring to the vector

(a)

M1 for any complete, valid method for obtaining $\overrightarrow{AB}$ in terms of **a** and **e**

A1 for $\overrightarrow{AB} = 2(\mathbf{a}+\mathbf{e})$ oe **must** be simplified.

(b)

M1 for any complete, valid method for obtaining $\overrightarrow{BE}$ in terms of **a** and **e**

A1 for $\overrightarrow{BE} = -(3\mathbf{a} + \mathbf{e})$ oe **must** be simplified.

(c)

M1 for any complete, valid method for obtaining $\overrightarrow{PC}$ in terms of **a** and **e.** Must include the correct use of the ratio.

A1 for a correct unsimplified expression for $\overrightarrow{PC}$ in terms of **a** and **e**

A1 for $\overrightarrow{PC} = \frac{6}{5}\mathbf{a} + \frac{11}{5}\mathbf{e}$ oe

(d)

M1 for obtaining $\overrightarrow{OQ}$ in terms of **a** and **e,** using the collinearity of *P*, *Q* and *C*

A1 for an unsimplified correct expression for $\overrightarrow{OQ}$ in terms of **a** and **e**

B1 for a second correct expression for $\overrightarrow{OQ}$ in terms of **a** and **e** using *O*, *E* and *Q*

M1 for equating components in the two expressions

A1 for a correct value for either of the 2 unknowns that were introduced

A1cao for deducing that $\lambda = \frac{21}{5},\ \mu = \frac{26}{5}$ need not be shown explicitly

*Alternative:*

| | |
|---|---|
| $\overrightarrow{PQ} = 2\overrightarrow{PC} = \frac{2}{5}(6\mathbf{a} + 11\mathbf{e})$ | B1Award when $\frac{2}{5}(6\mathbf{a} + 11\mathbf{e})$ seen |
| $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \mathbf{a} + \frac{4}{5}(\mathbf{a} + \mathbf{e}) + \frac{2}{5}(6\mathbf{a} + 11\mathbf{e})$ | M1A1 |
| $= \mathbf{a}\left(1 + \frac{4}{5} + \frac{12}{5}\right) + \mathbf{e}\left(\frac{4}{5} + \frac{22}{5}\right)$ | M1 |
| $\therefore \overrightarrow{OQ} = \frac{21}{5}\mathbf{a} + \frac{26}{5}\mathbf{e}\ \lambda = \frac{21}{5},\ \mu = \frac{26}{5}$ | A1A1 |

| Question Number | Answer | Marks |
|---|---|---|
| **9** | | |
| **(a)** | $(1-x)^{-k}=1+(-k)(-x)+\frac{(-k)(-k-1)}{2!}(-x)^2+\frac{(-k)(-k-1)(-k-2)}{3!}(-x)^3$ | M1 (1 needed; 2 or 2!, 6 or 3!) |
| | $=1+kx+\frac{k(k+1)}{2}x^2+\frac{k(k+1)(k+2)}{6}x^3$ * | A2,1,0 (algebraic terms) (3) |
| **(b)** | $(1+kx)^{\frac{1}{2}}=1+\frac{1}{2}kx+\frac{\frac{1}{2}(-\frac{1}{2})}{2!}(kx)^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(kx)^3$ | M1 |
| | $=1+\frac{1}{2}kx-\frac{1}{8}k^2x^2+\frac{1}{16}k^3x^3$ | A2,1,0 (3) |
| **(c)** | $-\frac{1}{8}k^2=\frac{k(k+1)}{2}$ | M1 |
| | $5k^2+4k=0$ | M1 |
| | $k=-\frac{4}{5}\quad k\neq 0$ | A1 (3) |
| **(d)** | $\sqrt{15}=\sqrt{\frac{3\times 25}{5}}=5\sqrt{\frac{3}{5}}$ | M1A1 (2) |
| | **Alt:**<br>$\sqrt{15}=\lambda\sqrt{\frac{3}{5}}\Rightarrow\sqrt{15\times\frac{5}{3}}=\lambda\quad\lambda=5$ | |
| **(e)** | $x=\frac{1}{2}$ | B1 |
| | $\sqrt{\frac{3}{5}}=\left(1-\frac{1}{2}\times\frac{4}{5}\right)^{\frac{1}{2}}=1-\frac{2}{5}\times\frac{1}{2}-\frac{1}{8}\times\frac{16}{25}\times\frac{1}{4}-\frac{1}{16}\times\left(\frac{4}{5}\right)^3\times\frac{1}{8}$ | M1A1 |
| | $\sqrt{15}=5\sqrt{\frac{3}{5}}=3.88$ | A1 (4) |
| | | [15] |

**Notes**

(a)

M1 for attempting a binomial expansion of $(1-x)^{-k}$. Must have the 1, 2 or 2! and 6 or 3!. It must be clear that $-x$ has been used in at least one term. This is a "show that" question, so simplifying all the terms immediately is insufficient method and gets M0

A1 for two correct algebraic terms

A1cso for all three algebraic terms correct. This is a given answer, so check working carefully.

(b)

M1 for attempting a binomial expansion of $(1+kx)^{\frac{1}{2}}$. Again must have the 1, 2 or 2! and 6 or 3!. It must be clear that $kx$ has been used in at least one term.

A1 for two correct algebraic terms **must** be simplified.

A1cso for all three algebraic terms correct **must** be simplified.

(c)

M1 for equating *their* coefficients of $x^2$ to form an equation. Allow if $x^2$ is included in both terms.

M1 for reducing *their* equation to a two term quadratic or linear equation.

A1cso for $k=-\frac{4}{5}$ ( $k \neq 0$ need not be seen)

(d)

M1 for manipulating either side of $\sqrt{15}=\lambda\sqrt{\frac{3}{5}}$ to obtain a value for $\lambda$

A1 for $\lambda = 5$ need not be shown explicitly.
If $\lambda = 5$ is seen w/o working, give M1A1

(e) B1 for identifying $x=\frac{1}{2}$ needed May only be seen in the expansion

M1 for substituting *their* values of $x$ and $k$ in their expansion from (b) to obtain a numerical expression for $\sqrt{\frac{3}{5}}$

A1 for an expansion which is fully correct, no need to evaluate here

A1cso for completing to $\sqrt{15}=5\sqrt{\frac{3}{5}}=3.88$

| Question Number | Answer | Marks |
| --- | --- | --- |
| **10** **(a)** | $ar + ar^2 = 7.5$ | M1 |
| | $S = \dfrac{a}{1-r} = 20$ | A1 |
| | $\dfrac{7.5}{r+r^2} = 20(1-r)$ | M1dep |
| | $3 = 8(1-r)(r+r^2) = 8(r-r^3)$ | |
| | $8r^3 - 8r + 3 = 0$ * | A1 (4) |
| **(b)** | $8\times\dfrac{1}{8} - 8\times\dfrac{1}{2} + 3 = 0$ | B1 (1) |
| **(c)** | $(2r-1)(4r^2+2r-3) = 0$ (or by division) | M1 |
| | $\left(r = \dfrac{1}{2}\right)$ $r = \dfrac{-2 \pm \sqrt{4-4\times4\times(-3)}}{8} = \dfrac{-2\pm\sqrt{52}}{8}, = 0.65...\ \ -1.15$ | M1,A1 |
| | $r = 0.65$ too big $r = -1.15$ not convergent<br>$\therefore$ only possible value for $r$ is $\dfrac{1}{2}$ | A1 (4) |
| **(d)** | $\dfrac{a}{1-\frac{1}{2}} = 20$ $\left(\text{or } a = \dfrac{7.5}{\frac{1}{2}\times\frac{3}{2}}\right)$ | M1 |
| | $a = 10$ | A1 (2) |
| **(e)** | 99% of 20 or $0.99\times 20$ or 19.8 seen | B1 |
| | $\dfrac{10(1-0.5^n)}{1-0.5} > 19.8$ | M1A1 |
| | $1-0.5^n > \dfrac{19.8}{20}$ $(=0.99)$ | |
| | $0.01 > 0.5^n$ | M1 |
| | Solve by logs to obtain $n > 6.6$ (or by trial and error) | M1 dep |
| | $n = 7$ | A1 (6)<br>[17] |

| | | |
|---|---|---|
| **(e)** | **Alt:** <br> $S_n = \dfrac{10\left(1-\left(\frac{1}{2}\right)^n\right)}{\frac{1}{2}}$ <br><br> $= 20 - 20\left(\frac{1}{2}\right)^n > 0.99\times 20$ <br><br> $20\left(\frac{1}{2}\right)^n < 0.01\times 20$ <br><br> $\left(\frac{1}{2}\right)^n < \frac{1}{100}$ <br><br> $2^n > 100$ <br><br> $\Rightarrow n = 7$ is least value <br> (Award M1 A0 if $n = 6.6$ seen) | <br><br><br><br> M1A1 <br> B1 (0.99x20) <br><br><br> M1 <br><br><br> M1depA1..(6) |

**Notes**

(a)

M1 for forming an equation using the given information - award for either equation. Formulae used must be correct

A1 for forming a second equation and both equations fully correct

M1dep for eliminating *a* between the two equations. The two equations do not need to be correct but the first M mark must have been gained.

A1cso for $8r^3 - 8r + 3 = 0$ *

(b)

B1 for substituting $r = \frac{1}{2}$ in the **given** equation and showing that this gives lhs = 0

There are longer methods. Provided the work shows that $r = \frac{1}{2}$ is a root of the equation, award B1.

(c)

M1 for using the factor $(2r-1)$ to factorise the equation either by inspection or division. This work may have been done in (b). If seen in (b) award this mark.

M1 for solving the quadratic by the formula or completing the square (see general principles for further information)

A1 for **both** values of *r* from the quadratic. One sf or surd form is sufficient here

A1ft for deducing that $r=\frac{1}{2}$ is the only possible value. Award this mark even if the values obtained from the quadratic are incorrect, providing they are **both** outside the range -1 < *r* < 0.6. If the range is stated to be 0 < *r* < 0.6 award A0.

(d)

M1 for using either of the equations formed in (a) with $r=\frac{1}{2}$ to obtain a value for *a*

A1cao for $a=10$

(e)

B1 for $99\%$ of 20 (or $0.99\times 20$ or 19.8 seen)

M1 for using the formula for the sum of the first *n* terms (formula must be correct) and setting up an inequality or equation with $r=\frac{1}{2}$, their *a* and their evaluated 99% of 20

A1 for a completely correct inequality or equation

M1 for solving to a 2 term inequality or equation with $\left(\frac{1}{2}\right)^n$ oe included

M1dep for solving *their* inequality or equation, logs can be used or trial and error. If logs used, with a correct inequality, expect to see *n* > 6.6 oe; if trial and error used expect to see indication that 6 is too small and 7 works (or too large if solving an equation). Dependent on **both** previous M marks.

A1cso for $n=7$. (Some candidates make two sign errors in their working. Such work can gain the M marks but scores A0 here as their solution is incorrect.)

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